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1984 J. Phys. A: Math. Gen. 17 2017

(http://iopscience.iop.org/0305-4470/17/10/014)

J. Phys. A: Math. Gen. 17 (1984) 2017–2026. Printed in Great Britain

Aspects of transformation to the space-independent frame for nonlinear wave processes in plasmas: I. Cold plasmas

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Received 24 March 1983, in final form 16 February 1984

Abstract. Transformation of nonlinear plasma equations to a space-independent frame with the help of the Lorentz transformation gives in place of partial differential equations a set of ordinary differential equations in a single independent variable for solution of nonlinear field equations. In this paper, this transformation has been used to yield the nonlinear precessional rotation of electromagnetic waves in plasmas in addition to the nonlinear shift in a wave parameter. Moreover, the Lagrangian and Hamiltonian of motion for a circularly polarised wave have been transformed to the space-independent frame and the equations of Akhiezer and Polovin for nonlinear plasma oscillations have been rectified by making them relativistically correct.

1. Introduction

Transformations of the field equations to a space-independent time-like coordinate system with the help of the Lorentz transformation (LT) can be very well used to solve higher-order differential equations for investigating nonlinear phenomena, e.g. self-action effects in plasmas. Winkles and Eldridge (1972) first used the LT to derive self-consistent solutions of the relativistic Vlasov–Maxwell equations, and found that a pure transverse wave cannot exist in such cases but a coupled longitudinal field necessarily appears. Analysing wave propagation in a cold, collisionless, two-component plasma in the frame of reference in which the field is not space independent, Clemmow (1974) showed that if there is no ambient magnetic field the solutions of the field equations will be those of ordinary second-order nonlinear differential equations. He also discussed the significance of stream velocity on the propagation of electromagnetic waves. Subsequently, Clemmow (1975, 1977), Chian and Clemmow (1975), Kennal and Pellat (1976), Shih (1978), Decoster (1978), Clemmow and Harding (1980) and Lee and Lerche (1978, 1979a, b, c, 1980) extended the usage of LT to investigate different types of nonlinear problems in various kinds of plasmas.

In the present paper, nonlinear space- and time-dependent field equations of an unbounded multicomponent plasma having electrons and ions (positive and negative) have been transformed to a purely time-dependent set of ordinary differential equations using the LT. Using the equations in the space-independent frame, the nonlinear effects investigated are the intensity-dependent self-precession of the polarisation of an electromagnetic wave and the shift in wavenumber. Earlier workers, however, used different formalisms, e.g. the BKM method (Chakraborty and Chandra 1978, Chandra 1980), the Lindstedt method (Chandra 1974, 1979) and some other available methods (Arons and Max 1974, Katz *et al* 1975, Lai and Wonnacott 1976, Khan and Chakraborty 1979, Chakraborty *et al* 1980, 1981, 1982, 1983, Bhattacharyya and Chakraborty 1979, 1982, Bhattacharyya 1981, 1983), to derive the expressions for precessional rotation and its complementary effects of electromagnetic waves in plasmas. In our present analysis we have also generalised some well known results reported earlier by Akhiezer and Polovin (1956) on electron motion in the presence of a strong electromagnetic wave. Circularly polarised waves being important in the study of strong waves in a medium, the exact solutions are obtained without any simplifying approximation and the Lagrangian and Hamiltonian of motions are also obtained for these waves. The mathematical techniques developed here may be useful in the study of other types of nonlinear problems in plasmas.

2. The basic assumptions and field equations

We assume (i) the plasma is stationary, cold and homogeneous; (ii) the incident electromagnetic wave is strong enough for the occurrence of relativistic motions of ions and electrons; (iii) the incident EM wave propagating along the z axis is transverse, circularly or elliptically polarised, and sinusoidal; (iv) the power of the wave is below a certain threshold limit so that self-focusing and self-trapping mechanisms are insignificant; (v) self-action effects arising from the ponderomotive force and thermal instabilities are negligible; (vi) the forces arising due to collision and gravitation are negligibly small in comparison with other forces present in the medium.

Under these assumptions, the field equations in the laboratory S-frame can be written as

$$(\partial/\partial t+\boldsymbol{v}_\alpha\cdot\nabla)\boldsymbol{p}_\alpha=q_\alpha[\boldsymbol{E}+(\boldsymbol{v}_\alpha\times\boldsymbol{B})/c], \tag{1}$$

$$(\partial/\partial t)N_\alpha+\operatorname{div}(N_\alpha\boldsymbol{v}_\alpha)=0, \tag{2}$$

$$\operatorname{curl}\boldsymbol{E}=-(1/c)\partial\boldsymbol{B}/\partial t, \tag{3}$$

$$\operatorname{curl}\boldsymbol{B}=(1/c)\partial\boldsymbol{E}/\partial t+(4\pi/c)\sum_\alpha N_\alpha\boldsymbol{v}_\alpha q_\alpha, \tag{4}$$

$$\operatorname{div}\boldsymbol{E}=4\pi\sum_\alpha q_\alpha N_\alpha, \tag{5}$$

$$\operatorname{div}\boldsymbol{B}=0 \tag{6}$$

with

$$\boldsymbol{p}_\alpha=m_{0\alpha}\boldsymbol{v}_\alpha/(1-v_\alpha^2/c^2)^{1/2} \tag{7}$$

where the index $\alpha=1,2,3$ (parameters with subscripts 1, 2 and 3 denote the electron, positive-ion and negative-ion components respectively); $q_1=-e$, $q_2=e$, $q_3=-e$; $m_{0\alpha}$ is the rest mass and other quantities have their usual meanings.

3. Transformations to the space-independent frame

3.1. Space-independent frame

The LT from the S frame to the S' frame moving with relative velocity V_0 parallel to the z axis is given by

$$x=x', \qquad y=y', \qquad z=\gamma_0(z'+V_0t'), \qquad t=\gamma_0(t'+V_0z'/c^2) \tag{8}$$

where

$$\gamma_0 = (1 - V_0^2/c^2)^{-1/2}. \tag{9}$$

Following Decoster (1978), if we put $\beta_0 = \tanh \psi_0$ in (8), we find that

$$z = z' \cosh \psi_0 + ct' \sin \psi_0, \qquad t = t' \cosh \psi_0 + (z'/c) \sinh \psi_0, \tag{10}$$

where ψ_0 is the hyperbolic angle for the S' system relative to the S system. Using (10) we obtain

$$\omega t - kz = (\omega \cosh \psi_0 - kc \sinh \psi_0)t' - [k \cosh \psi_0 - (\omega/c) \sinh \psi_0]z' \tag{11}$$

where k and ω are the constant wavenumber and wavefrequency respectively of an electromagnetic wave. Now following Winkles and Eldridge (1972), the velocity of the S' frame is assumed to be

$$V_0 = kc^2/\omega = c^2/V \tag{12}$$

where V is the phase velocity of the wave. Then (10) leads to

$$\omega t - kz = (\omega/\gamma_0)t' = \omega' T \tag{13}$$

where $\omega' = \omega/\gamma_0$ and t' is replaced by T. This relation immediately gives

$$\partial/\partial t = \gamma_0 \partial/\partial T, \qquad \partial/\partial z = -(\gamma_0/V)\partial/\partial T. \tag{14}$$

Relations (13) and (14) indicate that the LT enables us to change the variables from the space–time-dependent S frame to the space-independent S' frame of primed variables. For transverse waves $V > c$ and so $V_0 < c$; hence the relative velocity of S' and transformation relations of (13) and (14) are not unphysical. It is important to mention that Akhiezer *et al* (1975) considered the non-relativistic linear transformation rule

$$\omega t - kz = \omega T, \qquad V = \omega/k \tag{15}$$

to solve the plasma equations (1)–(6) to investigate some nonlinear effects, and Boyd and Sanderson (1969) took the special value

$$V = c \tag{16}$$

for some investigations.

3.2. *Transformation of some field variables*

The electric and magnetic field components of the S system can be written in the S' system as

$$\begin{aligned} E_x &= \gamma_0(E'_x + \beta_0 B'_y) = E'_x \cosh \psi_0 + B'_y \sinh \psi_0, \\ E_y &= \gamma_0(E'_y - \beta_0 B'_x) = E'_y \cosh \psi_0 - B'_x \sinh \psi_0, \\ E_z &= E'_z, \end{aligned} \tag{17}$$

$$\begin{aligned} B_x &= \gamma_0(B'_x - \beta_0 E'_y) = B'_x \cosh \psi_0 - E'_y \sinh \psi_0, \\ B_y &= \gamma_0(B'_y + \beta_0 E'_x) = B'_y \cosh \psi_0 + E'_x \sinh \psi_0, \\ B_z &= B'_z, \qquad \text{where } \beta_0 = V_0/c. \end{aligned} \tag{18}$$

The transformation of velocity components from the S frame to the S' frame is given by

$$(v_{\alpha x}, v_{\alpha y}) = \frac{(v'_{\alpha x}, v'_{\alpha y})}{\gamma_0(1+\beta_0 v'_{\alpha z}/c)} = \frac{c(v'_{\alpha x}, v'_{\alpha y})}{c\cosh\psi_0 + v'_{\alpha z}\sinh\psi_0},$$
$$v_{\alpha z} = \frac{(v'_{\alpha z}+\beta_0 c)}{(1+\beta_0 v'_{\alpha z}/c)} = \frac{c(v'_{\alpha z}\cosh\psi_0 + c\sinh\psi_0)}{c\cosh\psi_0 + v'_{\alpha z}\sinh\psi_0}. \tag{19}$$

The transformation of mass being

$$m_\alpha = m'_\alpha \gamma_0(1+\beta_0 v'_{\alpha z}/c),$$

as may be seen from Hughes and Young (1966) (where m_0 is the rest mass), the momentum components are transformed as

$$p_{\alpha x} = m_\alpha v_{\alpha x} = \frac{m'_\alpha \gamma_0(c+\beta_0 v'_{\alpha z})}{c\cosh\psi_0 + v'_z\sinh\psi_0} = m'_\alpha v'_{\alpha x} = p'_{\alpha x},$$
$$p_{\alpha y} = m_\alpha v_{\alpha y} = m'_\alpha v'_{\alpha y} = p'_{\alpha y},$$
$$p_{\alpha z} = m_\alpha v_{\alpha z} = \frac{m'_\alpha \gamma_0(c+\beta_0 v'_{\alpha z})(v'_{\alpha z}\cosh\psi_0 + c\sinh\psi_0)}{(c\cosh\psi_0 + v'_{\alpha z}\sinh\psi_0)} \tag{20}$$
$$= p'_{\alpha z}\gamma_0 + m'_\alpha c\beta_0\gamma_0 = \gamma_0(p'_{\alpha z} + m_0 V_0 \gamma'_\alpha).$$

Defining now the momentum-like quantities q and q' as

$$q_\alpha = (m_{0\alpha}^2 c^2 + p_\alpha^2)^{1/2}, \qquad q'_\alpha = (m_{0\alpha}^2 c^2 + p'^2_\alpha)^{1/2}, \tag{21}$$

we find that

$$q'^2_\alpha = m_{0\alpha}^2 c^2 + \frac{m_{0\alpha}^2 V'^2_\alpha}{(1-V'^2/c^2)} = \frac{m_{0\alpha}^2 c^2}{1-V'^2/c^2} = m'^2_\alpha c^2 \tag{22}$$

where V' is the velocity of the S' system relative to the rest system S_0. So we have

$$p_{\alpha z} = p'_{\alpha z}\cosh\psi_0 + q'_\alpha\sinh\psi_0,$$
$$q_\alpha = m_\alpha c = m'_\alpha\gamma_0 c(1+\beta_0 v'_{\alpha z}/c) = m'_\alpha\gamma_0(c+\beta_0 v'_{\alpha z}) \tag{23}$$
$$= \gamma_0(q'_\alpha + p'_{\alpha z}\beta_0) = q'_\alpha\cosh\psi_0 + p'_{\alpha z}\sinh\psi_0.$$

If N, N', N_0 are the symbols for the number density in the three systems S, S', S_0 respectively, then again following Hughes and Young (1966) we have

$$N'_\alpha = \frac{N_{0\alpha}}{(1-V'^2/c^2)^{1/2}}, \qquad m_{0\alpha}N'_\alpha = \frac{m_{0\alpha}N_{0\alpha}}{(1-V'^2/c^2)^{1/2}} = m'_\alpha N_{0\alpha}, \tag{24}$$
$$N'_\alpha/N_\alpha = m'_\alpha/m_\alpha = m'_\alpha c/m_\alpha c = q'_\alpha/q_\alpha.$$

The first relation of (24) gives

$$N_{0\alpha} = N'_\alpha \operatorname{sech}\psi' = \text{constant} \tag{25}$$

and the third relation of (24) can be written as

$$N_\alpha = \frac{N'_\alpha q_\alpha}{q'_\alpha} = \frac{N_{0\alpha} q_\alpha \cosh\psi'}{[q_\alpha\cosh(\psi'-\psi) - p_{\alpha z}\sinh(\psi'-\psi)]} \tag{26}$$

where ψ is the hyperbolic angle for the S system relative to the S_0 frame and ψ' is the same for the S' system. Therefore, in the S system the number density is not constant, and in the absence of the negative ions, electron and positive-ion number densities are not necessarily equal.

For the vector and scalar potentials A and ϕ, since these potentials form a four-vector, we can write

$$A_x = A'_x, \qquad A_y = A'_y, \qquad A_z = \gamma_0(A'_z + \beta_0\phi') = A'_z \cosh\psi_0 + \phi' \sinh\psi_0,$$
$$\phi = \gamma_0(\phi' + \beta_0 A'_z) = \phi' \cosh\psi_0 + A'_z \sinh\psi_0. \tag{27}$$

3.3. *Transformation of the field equations to the space-independent frame*

We assume that there is no static magnetic field in the plasma and so $B_z = 0 = B'_z$. Therefore, in the S' frame equation (1) becomes

$$\frac{\partial}{\partial T} p'_{\alpha\pm} = q_\alpha E'_\pm \mp \frac{\mathrm{i} q_\alpha B'_\pm V}{c(\gamma_0 V - V_0)} [V_0(\gamma_0 - 1) - V\gamma_0^{-2}\gamma'^{-1}_{\alpha v} v'_{\alpha z}], \tag{28}$$

$$\begin{aligned}\gamma'_{\alpha v}\frac{\partial p'_{\alpha z}}{\partial T} = {} & q_\alpha V E'_z(1 + p'_{\alpha z}/m_{0\alpha}\gamma_0\gamma'_\alpha V_0) + \frac{q_\alpha(p'_{\alpha +}E'_- + p'_{\alpha -}E'_+)}{2m_{0\alpha}\gamma'_\alpha V_0(V + v'_{\alpha z})} \\ & \times\{V(\gamma_0^2 V - \gamma_0 V_0 - V_0) + v'_{\alpha z}[V(\gamma_0^2 - \gamma_0 + 1) - V_0]\} \\ & + \frac{\mathrm{i} q_\alpha(p'_{\alpha +}\Omega'_{\alpha -} - p'_{\alpha -}\Omega_{\alpha +})V^2}{\gamma_0^4\gamma'_\alpha V_0(V + v'_{\alpha z})(\gamma_0 V - V_0)} \\ & \times[(V + V\gamma_0^6 - V_0\gamma_0^5)v'_{\alpha z} + \gamma_0^4(\gamma_0 V - V_0)^2],\end{aligned} \tag{29}$$

where

$$p'_{\alpha\pm} = p'_{\alpha x} \pm \mathrm{i} p'_{\alpha y}, \qquad E'_\pm = E'_x \pm \mathrm{i} E'_y, \qquad \Omega'_{\alpha\pm} = q_\alpha B'_\pm / m_{0\alpha} c,$$
$$B'_\pm = B'_x \pm \mathrm{i} B'_y, \qquad \gamma'_{\alpha v} = \gamma_0[\gamma_0 V - V_0 + v'_{\alpha z}(\gamma_0 - 1)],$$

and parameters with plus and minus signs indicate the values for the LCP (left circular polarisation) and RCP (right circular polarisation) components of the electromagnetic wave.

The equation of continuity (2) yields

$$N'_\alpha = N_{0\alpha} = \text{constant}. \tag{30}$$

From (3) we obtain

$$B'_x = \text{constant}$$

similarly $\qquad B'_y = \text{constant}.$

Moreover, (4) gives

$$\frac{\partial}{\partial T}(E'_\pm) = -\sum_\alpha \frac{m_{0\alpha}\omega^2_{p\alpha} v'_{\alpha\pm}}{q_\alpha(1 + \beta_0\beta'_{\alpha z})}; \qquad v'_{\alpha\pm} = v'_{\alpha x} \pm \mathrm{i} v'_{\alpha y}, \tag{32}$$

$$\frac{\partial}{\partial T}(E'_z) = -\sum_\alpha \frac{m_{0\alpha}\omega^2_{p\alpha}(v'_{\alpha z} + V_0)}{\gamma_0 q_\alpha(1 + \beta_0\beta'_{\alpha z})}. \tag{33}$$

Since $v_{\alpha z} = 0$ when $v'_{\alpha z} = -V_0$, we may put $v_{\alpha z} = -V_0 + \delta v'_{\alpha z}$ to avoid a physical impossibility where $\delta v'_{z\alpha}$ are the second-order velocity components parallel to OZ'. So expanding $(1+\beta_0\beta'_{\alpha z})^{-1}$ and neglecting higher powers of $\beta'_{\alpha z}$ we obtain

$$N_\alpha = N_{0\alpha}\gamma_0^2\delta v'_{\alpha z}/V, \tag{34}$$

$$\partial(E'_\pm)/\partial T = -\sum \gamma_0 m_{0\alpha}\omega_{p\alpha}^2 v'_{\alpha\pm}/q_\alpha, \tag{35}$$

$$\partial(E'_z)/\partial T = -\sum (\gamma_0 m_{0\alpha}\omega_{p\alpha}^2/q_\alpha)\delta v'_{\alpha z}. \tag{36}$$

4. Nonlinear shift in wave parameter and precessional rotation of an electromagnetic wave

4.1. The case of a circularly polarised wave

For a circularly polarised wave, we put

$$E'_\pm = a e^{\pm i\theta} \qquad \text{and} \qquad E'_z = 0 \tag{37}$$

where a is the amplitude of the wave and $\theta = \omega'_\pm T$. Therefore, for such a wave, nonlinear dispersion relations for LCP and RCP components are obtained from (28)–(36) as

$$\omega'^2_\pm = \sum \omega_{p\alpha}^2[1-(\eta_{a\alpha}^2/\omega_{p\alpha}^2)\omega'^2_\pm]^{1/2} \tag{38}$$

where

$$\eta_{a\alpha} = (q_\alpha a/m_{0\alpha}\omega_{p\alpha}c).$$

Replacing $\omega'_\pm$ by $\omega_\pm/\gamma_0$ in (38), the exact dispersion relation can be obtained for the laboratory frame. After expanding the square root term we recover the modified form of the equation of Arons and Max (1974) for multicomponent plasmas, if $\eta_{a\alpha}^4$ are neglected. To find the nonlinear wavenumber shift of the electromagnetic wave we put $\omega_\pm = \omega$ and $k_\pm = k + \delta k_\pm$ in the dispersion relation, obtained from (38), for the laboratory frame. $\delta k_\pm$ are the nonlinear increments in wavenumber. Thus the wavenumber shifts become

$$\delta k_\pm/k = \sum \eta_{a\alpha}^2 x_{p\alpha}/4(1-x_{p\alpha}). \tag{39}$$

This relation shows that the LCP and RCP components of the wave have the same wavenumber shift. So the precessional rotation $\phi[=\frac{1}{2}(\delta k_- - \delta k_+)z]$ becomes zero.

4.2. The case of an elliptically polarised wave

Since an elliptically polarised wave is a combination of two circularly polarised waves, we write

$$E'_\pm = \tfrac{1}{2}[(a\pm b)e^{i\theta_0} + (a\mp b)e^{-i\theta_0}] = a\cos\theta_0 \pm ib\sin\theta_0 \tag{40}$$

where $\theta_0 = \omega'_\pm T$, a and b being the amplitudes of two circularly polarised waves.

Therefore, from (28)–(36) finding the first harmonic solution for the third-order fields, the nonlinear dispersion relation becomes

$$\omega'^2_\pm = \sum \{\omega_{p\alpha}^2 + k_\pm^2c^2(\eta_{a\alpha}\pm\eta_{b\alpha})^2/2(4-x_{p\alpha\pm}) - \tfrac{1}{8}\omega_\pm^2[3(\eta_{a\alpha}^2+\eta_{b\alpha}^2)\mp 2\eta_{a\alpha}\eta_{b\alpha}]\} \tag{41}$$

where

$$\eta_{a\alpha} = (q_\alpha a/m_{0\alpha}\omega c), \qquad \eta_{b\alpha} = (q_\alpha b/m_{0\alpha}\omega c).$$

In the laboratory frame, (41) becomes

$$n_{\pm}^{2}-1+\sum\left[x_{p\alpha}-(\eta_{a\alpha}\mp\eta_{b\alpha})n_{\pm}^{2}/2(4-x_{p\alpha\pm})\right.$$
$$\left.-\tfrac{3}{8}x_{p\alpha\pm}(\eta_{a\alpha}^{2}+\eta_{b\alpha}^{2})\pm\tfrac{1}{4}x_{p\alpha\pm}(\eta_{a\alpha}\eta_{b\alpha})\right]=0 \tag{42}$$

where

$$n_{\pm}=(k_{\pm}c/\omega_{\pm}).$$

Putting $k_{\pm}=k+\delta k_{\pm}$ and $\omega_{\pm}=\omega$ in (42), we obtain

$$\frac{\delta k_{+}+\delta k_{-}}{2k}=\sum x_{p\alpha}\frac{(\eta_{a\alpha}^{2}+\eta_{b\alpha}^{2})}{4}\left(\frac{1-x_{p\alpha}}{4-x_{p\alpha}}\right)-\tfrac{3}{4}, \tag{43}$$

$$\frac{\delta k_{-}-\delta k_{+}}{2k}=\sum\frac{x_{p\alpha}\eta_{a\alpha}\eta_{b\alpha}}{2}\left(\frac{1-x_{p\alpha}}{4-x_{p\alpha}}\right)-\tfrac{1}{4}. \tag{44}$$

The expression (43) gives the average of the wavenumber shifts of LCP and RCP components of the wave. Expression (44) shows that an elliptically polarised wave gives rise to precessional rotation (PR) without changing the ellipticity of the polarisation ellipse. Expressions (43) and (44) are modified forms of formulae (24) and (25) of Arons and Max (1974) for multicomponent plasmas if their symbols are used. Moreover, if we put $\eta_{b\alpha}=0$ in (43) and neglect the contributions of positive and negative ions of the plasmas, we recover the results for the wavenumber shift of a plane polarised wave, first obtained by Sluijter and Montgomery (1965).

5. Akhiezer- and Polovin-type nonlinear equations

Putting $Q'_{\alpha}=p'_{\alpha}/m_{0\alpha}c$, and using (32) in the derivative of (28), we obtain for purely transverse vibrations

$$\partial^{2}Q'_{\alpha\pm}/\partial T^{2}+\left(\sum\gamma_{0}\omega_{p\alpha}^{2}\right)\beta'_{\alpha\pm}=0. \tag{45}$$

Similarly, for purely longitudinal oscillation we obtain

$$\frac{\partial}{\partial T}\left(\frac{\gamma'_{v}Q'_{\alpha z}}{V(1+Q'_{\alpha z}/\gamma_{0}^{2}\gamma'_{\alpha}\beta_{0})}\right)+\left(\sum\frac{\omega_{p\alpha}^{2}}{\gamma_{0}}\right)\frac{(\beta'_{\alpha z}+\beta_{0})}{(1+\beta_{0}\beta'_{\alpha z})}=0. \tag{46}$$

Equations (45) and (46), obtained for no static magnetic field, are relativistically correct and more general than equations (8.1.2.14) of Akhiezer *et al* (1975).

6. The Lagrangian and Hamiltonian in the space-independent frame

Following Landau and Lifshitz (1975) the Lagrangian and Hamiltonian in plasmas in the laboratory frame S are

$$\mathscr{L}=-\sum N_{0\alpha}\left[m_{0\alpha}c^{2}\left(1-\frac{v_{0\alpha}^{2}}{c^{2}}\right)^{1/2}-\frac{q_{\alpha}}{c}(\boldsymbol{A}\cdot\boldsymbol{v}_{\alpha})-q_{\alpha}\phi\right]+(E^{2}-B^{2})/8\pi \tag{47}$$

and

$$\mathscr{H}=\sum N_{0\alpha}\boldsymbol{v}_\alpha\left[m_{0\alpha}\boldsymbol{v}_\alpha\left(1-\frac{v_\alpha^2}{c^2}\right)^{-1/2}+\frac{q_\alpha}{c}\boldsymbol{A}\right]-\mathscr{L} \tag{48}$$

where

$$\boldsymbol{B}=\operatorname{curl}\boldsymbol{A},\qquad \boldsymbol{E}=-\frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t}+\boldsymbol{\nabla}\phi,\qquad \operatorname{div}\boldsymbol{A}=-\frac{1}{c}\frac{\partial\phi}{\partial t}.$$

In the S' frame, $\mathscr{L}$ and $\mathscr{H}$ are given by

$$\begin{aligned}\mathscr{L}'=&-\sum N_{0\alpha}\left\{m_{0\alpha}c^2\left[1-\frac{\gamma_0^2}{c^2}(v'_{\alpha+}v'_{\alpha-}+\gamma_0^2v'^2_{\alpha z})\left(1-\frac{\gamma_0^2}{V}v'_{\alpha z}+\frac{\gamma_0^4v'^2_{\alpha z}}{V^2}-\cdots\right)^2\right]^{1/2}\right.\\&+e\gamma_0\left(1-\frac{\gamma_0^2}{V}v'_{\alpha z}+\frac{\gamma_0^4}{V^2}v'^2_{\alpha z}-\cdots\right)[\tfrac{1}{2}(A'_+v'_{\alpha-}+A'_-v'_{\alpha+})+\gamma_0^2v'_{\alpha z}(A'_z+\beta_0\phi')]\\&\left.+e\gamma_0(\phi'+\beta_0A'_z)\right\}+\frac{\gamma_0^2(1-\beta_0^2)(E'_+E'_-)}{8\pi}+E'^2_z,\end{aligned} \tag{49}$$

$$\begin{aligned}\mathscr{H}'=&\sum N_{0\alpha}\left\{m_{0\alpha}c^2\left[1-\frac{\gamma_0^2}{c^2}(v_{\alpha+}v_{\alpha-}+\gamma_0^2v'^2_{\alpha z})\left(1-\frac{\gamma_0^2}{V}v'_{\alpha z}+\frac{\gamma_0^4}{V^2}v'^2_{\alpha z}-\cdots\right)^2\right]^{-1/2}\right.\\&\left.+e\gamma_0(\phi'+\beta_0A'_z)\right\}-\gamma_0^2(1-\beta_0^2)(E'_+E'_-)/8\pi-E'^2_z.\end{aligned} \tag{50}$$

Using (37), for a circularly polarised wave, (49) and (50) become

$$\mathscr{L}'=\sum\left[N_{0\alpha}m_{0\alpha}c^2\left(1-\frac{e^2a^2\omega'^2_\pm}{\omega^4_{p\alpha}m^2_{0\alpha}c^2}\right)^{1/2}+\frac{e^2a^2}{m^2_{0\alpha}\omega^2_{p\alpha}}\right]+\frac{\gamma_0^2a^2(1-c^2/V^2)}{8\pi} \tag{51}$$

and

$$\mathscr{H}'=\sum\left[N_{0\alpha}m_{0\alpha}c^2\left(1-\frac{e^2a^2\omega'^2_\pm}{\omega^4_{p\alpha}m^2_{0\alpha}c^2}\right)^{-1/2}\right]-\frac{\gamma_0^2a^2}{8\pi}\left(1-\frac{c^2}{V^2}\right). \tag{52}$$

The results for the Lagrangian and Hamiltonian in the laboratory frame are obtained simply by changing $\omega_\pm$ to $\omega_\pm/\gamma_0$ in (51) and (52) where $\gamma_0^2=\omega_\pm^2/(\omega_\pm^2-k_\pm^2c^2)$.

Following Whitham (1967, 1974), Dysthe (1974) and Das and Sihi (1979, 1980), the Lagrangian derived in (49) can be used for finding the nonlinear effect including shifts of the wave parameters in the space-independent frame with the help of the transformation relations of § 3.

7. Remarks

(1) To obtain the final results for shifts of the wave parameters and PR in the standard form one should transform the field variables at the beginning and then proceed with the calculation. If instead the equations were solved in the laboratory system before the transformation, and transformation relations used only in the subsequent stages, then the correct formulae would not be obtained in the space-independent frame.

(2) This technique of transformation to the space-independent frame is applicable only to a single propagating wave, because the condition required for space indepen-

dence, $V_0 = c^2/V = kc^2/\omega$, is not satisfied for two or more waves interacting with a medium.

(3) For very high intensity focused laser beams the PR will be seriously disturbed by the growth of inhomogeneity due to the local increase of the refractive index leading to filamentation, wave bunching etc. So one should first study instability of electromagnetic waves in the space-independent frame before evaluating the PR.

(4) By transforming the field equations to the appropriate space-independent frame, some of the field variables become either constant or zero (for example, the number density becomes constant and the oscillating magnetic field vanishes); for these reasons and since the differential equations become ordinary rather than partial differential equations in the laboratory frame, some of the nonlinear terms vanish. But as far as ease of manipulation is concerned the net advantage may not be as great as at first sight because complications arise due to the LT and the consequent velocity transformation.

(5) When the interesting additional effects of collision, gravitation, kinetic temperature and static magnetic field etc are considered, evaluation of the nonlinear effects like precessional rotation and shift in a wave parameter with the help of LT will be, of course, complicated. Our work on these problems will be communicated elsewhere shortly.

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